A) THE PRODUCT RULE

When two expressions are multiplied together, the two brackets of the product could be expanded and the results differentiated term by term. In many cases this expansion would be difficult or tedious. A useful rule to deal with this is the product rule.

In the product rule, the two expressions are treated as individual functions. For example, the product $(x + 4)(2x - 5)$ would be $f(x) = x + 4$ and $g(x) = 2x - 5$.

The product rule: If $y = f \cdot g$ then $y' = f' \cdot g + g' \cdot f$
that is, the derivative of the first function multiplied by the second function, plus the derivative of the second function multiplied by the first function.

Examples:

1) $y = (x + 4)(2x - 5)$

$$y' = 1 \cdot (2x - 5) + 2 \cdot (x + 4)$$
$$= 2x - 5 \quad + \quad 2x + 8$$
$$= 4x + 3$$

2) $y = 5x^3 \cos x$

$$f = 5x^3,\ g = \cos x$$
$$y' = 15x^2 \cdot \cos x \ + \ {}^{-}\sin x \cdot 5x^3$$
$$y' = 15x^2 \cos x - 5x^3 \sin x$$

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B) THE QUOTIENT RULE

If one expression is divided by a second expression, this is a quotient. To differentiate, we use the quotient rule.

The quotient rule: If $y = \frac{f}{g}$ then $y' = \frac{f' \cdot g - g' \cdot f}{g^2}$.

Examples:

1) $y = \frac{3x^2}{e^x}$

$$f = 3x^2,\ g = e^x$$
$$y' = \frac{6x \cdot e^x - e^x \cdot 3x^2}{(e^x)^2}$$

2) $y = \frac{2e^{5x}}{3x^2 - 4}$

$$y' = \frac{10e^{5x} \cdot (3x^2 - 4) - 6x \cdot 2e^{5x}}{(3x^2 - 4)^2}$$

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